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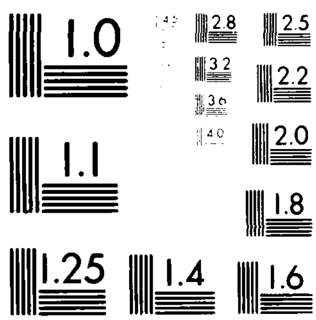

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Cambridge, MA 02139

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ON THE CALCULATION OF POTENTIAL FLOW ABOUT A BODY IN AN UNBOUNDED FLUID

by

F. Noblesse & G. Triantafyllou

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This study is concerned with the problem of calculating potential flow about a nonlifting body in an unbounded fluid. Several simple explicit approximations for the velocity potential are obtained and investigated numerically. Results of calculations are presented for the simple cases of potential flows due to translations of ellipsoids and ogives.

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ON THE CALCULATION OF POTENTIAL FLOW
ABOUT A BODY IN AN UNBOUNDED FLUID

Table of Contents

ABSTRACT

This study is concerned with the problem of calculating potential flow about a nonlifting body in an unbounded fluid. Several simple explicit approximations for the velocity potential are obtained and investigated numerically. Results of calculations are presented for the simple cases of potential flows due to translations of ellipsoids and ogives.

1. Introduction

A classical and fundamental problem in hydrodynamics is that of calculating potential flow due to motion of an arbitrary body through an unbounded ideal (incompressible and inviscid) fluid. This potential-flow problem indeed is directly relevant to naval hydrodynamics (e.g. calculation of flow about a deeply-submerged body and about a ship in slow motion) and to subsonic aerodynamics. Mathematically, the problem consists in determining the velocity potential (ϕ) given by the solution of the Laplace equation $\nabla^2\phi=0$ in the flow domain (d) exterior to the body surface (b), subject to the condition that ϕ vanishes "at infinity" (far away from the body) and to the Neumann condition that the normal derivative ϕ_n of the potential is prescribed on the body surface.

A great variety of methods have been developed for solving the above-defined exterior Neumann boundary value problem. These methods can be divided into two basic classes of methods, namely integral-equation methods in which an integral equation is formulated and solved numerically, and direct numerical methods, mainly of the finite-element type; hybrid methods, in which finite elements and an integral representation are coupled, however have also been used, e.g. by Jami and Lenoir (1977). Integral-equation methods are widely favored, as is apparent from the long list of references in the review of Hess (1975) and Körner and Hirschel (1977), and indeed are natural and a-priori advantageous methods in the sense that they explicitly take advantage of the Laplace equation for reducing the original three-dimensional problem to a two-dimensional computational problem in which the velocity potential only needs to be calculated on the body surface.

One of the earliest and most-widely used integral-equation methods is that of Hess and Smith (1966), in which an auxiliary distribution of sources on the body surface is employed, in the classical manner described by Kellogg (1929). Modifications of this method have been devised by Landweber and Macagno (1969), whose method mainly differs from that of Hess and Smith in the treatment of the singularity of the kernel of the integral equation and in the procedure for obtaining numerical solutions, and by Webster (1975), whose method is based on a Fredholm integral equation of the first kind obtained by assuming a distribution of sources on a surface slightly inside

the body surface. Methods based on the use of an auxiliary distribution of normal dipoles (instead of sources) on the body surface have also been used, e.g. by Chang and Pien (1975). Finally, methods based on an integral equation for the velocity potential itself (rather than for assumed auxiliary distributions of sources or dipoles as in the previous methods) have recently been used by Chow, Hou, and Landweber (1976), and by Noblesse for the analogous but more complex problems of ship wave resistance (1978) and of flow about a body in regular water waves (1980).

In this study, a new basic integral identity is presented. An interesting feature of this identity is that it is valid in the whole space (that is, both in the flow domain and inside the body, as well as exactly on the body surface), and is in fact equivalent to a set of three classical integral identities that are exclusively valid for points outside the body, exactly on the body, and inside the body. In the particular case when the Laplace equation is satisfied, this "unified integral identity" becomes identical to the integral equation of Chow, Hou, and Landweber (1976). This integral equation is first investigated by considering the simple cases of potential flows due to translations of elliptical cylinders and ellipsoids. The investigation then is continued for other, complementary, simple cases, namely that of potential flows due to translations of ogives. In particular, several simple explicit approximations for the velocity potential, which are suggested by the above-mentioned study of flows about ellipses and ellipsoids and indeed are exact for these particular flows, are evaluated numerically and compared to the exact potential (given analytically by conformal mapping). Finally, the basic integral equation of Landweber is expressed in a modified form, also of the nonhomogeneous Fredholm type. The given term in this modified nonhomogeneous Fredholm integral equation provides another explicit approximation for the potential. This explicit approximation is also exact in the particular cases of translations of ellipsoids, and it is compared to the exact potential for translations of ogives. The calculations for translations of ogives show that fairly-good approximations to the velocity potential can be obtained by the above-mentioned simple explicit approximations, notably the last one. For practical purposes, the main results of the present study indeed reside in these simple explicit approximations.

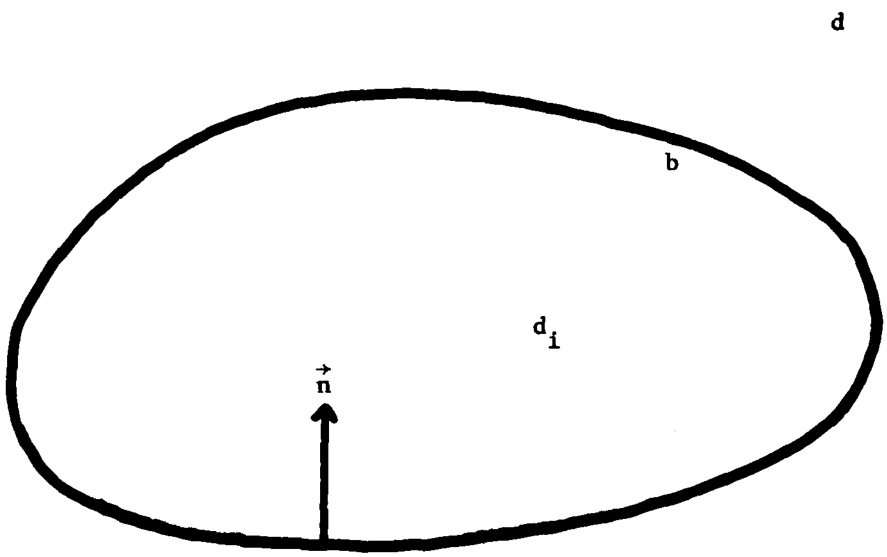

Figure 1 - Definition sketch

2. Basic integral identities

Throughout this study, variables are supposed to be nondimensional. It is well known that fundamental integral identities can be obtained for the velocity potential by applying a classical Green identity to the potential function $\phi(\vec{x})$ and the free-space Green function $G(\vec{x},\vec{\xi})$, which is given by

$$G(\vec{x},\vec{\xi}) = -1/4\pi[(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2]^{1/2} \;. \tag{2.1}$$

This classical Green identity is

$$\int_d (\phi\nabla^2 G - G\nabla^2\phi)\,dv = \int_b (\phi G_n - G\phi_n)\,da, \tag{2.2}$$

where the fact that $\phi=O(1/r)$ and $G=O(1/r)$ as $r\equiv(x^2+y^2+z^2)^{1/2}\to\infty$ was used to discard the integral of $\phi G_n - G\phi_n = O(1/r^3)$ over a large surrounding sphere of radius r (surface area $\sim r^2$). In formula (2.2), and indeed hereafter in this study, the following notation is used: $\phi\equiv\phi(\vec{x})$, $G\equiv G(\vec{x},\vec{\xi})$, $\nabla\equiv(\partial_x,\partial_y,\partial_z)$, $G_n\equiv\partial G/\partial n\equiv\nabla G\cdot\vec{n}$ and $\phi_n\equiv\partial\phi/\partial n\equiv\nabla\phi\cdot\vec{n}$, where $\vec{n}\equiv\vec{n}(\vec{x})$ is the unit normal vector, at point $\vec{x}$, to the body surface b pointing inside the body (outside the flow domain d), as is shown in figure 1; furthermore, $dv\equiv dv(\vec{x})$ and $da\equiv da(\vec{x})$ represent the differential elements of volume and area at point $\vec{x}$ of d and b, respectively.

The Green function $G(\vec{x},\vec{\xi})$ satisfies the well-known equation

$$\nabla^2 G = \delta(x-\xi)\delta(y-\eta)\delta(z-\zeta) \quad , \tag{2.3}$$

so that we have $\phi(\vec{x})\nabla^2 G\equiv\phi(\vec{\xi})\nabla^2 G+[\phi(\vec{x})-\phi(\vec{\xi})]\nabla^2 G\equiv\phi(\vec{\xi})\nabla^2 G$, provided ϕ is continuous in d+b, which is assumed here. Equation (2.3) then becomes

$$C\phi_* = \int_d G\nabla^2\phi\,dv - \int_b (G\phi_n - \phi G_n)\,da \quad , \tag{2.4}$$

where the notation $\phi_*\equiv\phi(\vec{\xi})$ was used for shortness (and will indeed be used hereafter in this study), and C is defined as

$$C = \int_d \nabla^2 G dv \quad . \tag{2.4a}$$

It follows from equation (2.3) that we have $C \equiv 1$ if the point $\vec{\xi}$ is in the exterior domain d strictly outside the body surface b, whereas we have $C \equiv 0$ if $\vec{\xi}$ is in the interior domain d_i strictly inside b.

We thus obtain the classical integral identities.

$$\phi_* = \int_d G\nabla^2\phi dv - \int_b (G\phi_n - \phi G_n) da \tag{2.5a}$$

for $\vec{\xi}$ in d-b, and

$$0 = \int_d G\nabla^2\phi dv - \int_b (G\phi_n - \phi G_n) da \tag{2.5b}$$

for $\vec{\xi}$ in d_i-b. It can also be seen from equation (2.3) that we have $C=1/2$ if the point $\vec{\xi}$ is exactly on the body surface b, at least for points $\vec{\xi}$ where the surface b is smooth; more generally, the value of $4\pi C$ at a point $\vec{\xi}$ of b is equal to the solid angle under which the exterior domain d is viewed from $\vec{\xi}$. We then have

$$\frac{1}{2}\phi_* = \int_d G\nabla^2\phi dv - \int_b (G\phi_n - \phi G_n) da \tag{2.5c}$$

for $\vec{\xi}$ exactly on (smooth) b. Identities (2.5a) and (2.5c) are usually obtained by applying the Green identity (2.2) to the domain d-ε, where ε represents the domain inside a small sphere centered at the point $\vec{\xi}$, or the intersection domain of this sphere with d if $\vec{\xi}$ is exactly on the boundary surface b; we then have $\nabla^2 G \equiv 0$ in d-ε, while on the right side of equation (2) there is an additional integral over the surface of the small sphere (or portion thereof) whose value can be shown to be equal to $-\phi_*$(or $-\phi_*/2$ if $\vec{\xi}$ is on b) in the limit as the radius of the sphere vanishes. This traditional derivation and the derivation used in this study, based on equation (2.3), are of course equivalent.

As is explicitly indicated in equations (2.5a,b,c), the value of the constant C on the left side of equation (2.4) is discontinuous across the body surface, C being equal to 1 outside the body and to 0 inside. This discontinuity in the value of C on the left side of equation (2.4) evidently is accompanied by a corresponding discontinuity on the right side of the equation. Specifically, the latter discontinuity stems from the integral $\int_b \phi G_n da$ representing the potential induced by a surface distribution of normal doublets of strength ϕ over b, as is well known. An integral identity valid for any point $\vec{\xi}$ -- either outside, inside or exactly on the body surface -- can be obtained by eliminating the discontinuity in the value of C in equation (2.4). This can readily by achieved by adding the term $C_i \phi_*$ on both the left and right sides of equation (2.4) with C_i given by

$$C_i = \int_{d_i} \nabla^2 G dv \equiv - \int_b G_n da \quad , \tag{2.6}$$

as may be obtained by using the divergence theorem. Equation (2.4) then becomes

$$\phi_* = \int_d G \nabla^2 \phi dv - \int_b [G\phi_n - (\phi - \phi_*) G_n] da \tag{2.7}$$

since we have $C+C_i=1$. The integral identity (2.7) is valid for any point $\vec{\xi}$ either outside, inside, or exactly on the body surface. This new identity thus is essentially equivalent to the set of the three classical identities (2.5a,b,c), which are valid exclusively for $\vec{\xi}$ outside, inside, or on the body surface, respectively. As a matter of fact, these three identities can be obtained from the identity (2.7) by noting that we have $-\int_b G_n da \equiv \int_{d_i} \nabla^2 G dv = 0, 1,$ or $1/2$ for $\vec{\xi}$ outside, inside, or exactly on the body.

Integral identities analogous to identities (2.5a,b,c) and 2.7) can evidently also be obtained for the "interior potential-flow problem". These identities are listed in appendix A. Another classical integral identity, obtained by combining these basic identities for the exterior and interior problems, is also given in the appendix.

3. Basic integral equation of potential flow about a body

In the particular case of potential flow about a body, we have $\nabla^2\phi=0$ in the (fluid) domain d outside the body, and the integral identity (2.7) becomes

$$\phi_* = -\int_b G\phi_n da + \int_b (\phi-\phi_*)G_n da \quad , \tag{3.1}$$

which is an integral equation for determining the velocity potential ϕ on the surface b of the body. More precisely, equation (3.1) is a nonhomogeneous (second kind) Fredholm integral equation, of the form

$$\phi(\vec{\xi}) = f(\vec{\xi}) + L(\vec{\xi};\phi) \quad , \tag{3.2}$$

where $f(\vec{\xi})$ is the known (since ϕ_n is specified on b) nonhomogeneous term given by

$$f(\vec{\xi}) = -\int_b G(\vec{x},\vec{\xi})\phi_n(\vec{x})da(\vec{x}) \quad , \tag{3.2a}$$

and $L(\vec{\xi};\phi)$ is the linear transform of ϕ defined by the integral

$$L(\vec{\xi};\phi) = \int_b [\phi(\vec{x})-\phi(\vec{\xi})]G_n(\vec{x},\vec{\xi})da(\vec{x}) \quad . \tag{3.2b}$$

The above integral equation has previously been obtained by Chow, Hou, and Landweber (1976), equation (19), by using a technique due to Landweber for removing the singularity in the dipole integral in the classical integral equation

$$\frac{1}{2}\phi_* = -\int_b G\phi_n da + \int_b \phi G_n da \tag{3.3}$$

given by the Green identity (2.5c) in the particular case when $\nabla^2\phi\equiv 0$ in d. The manner in which the integral equation (3.2) was obtained in the present study supplements the Landweber derivation and provides additional insight into this integral equation. In particular, it has been shown that equation (3.2) holds not only for $\vec{\xi}$ on the surface b of the body, but also for $\vec{\xi}$ in the fluid domain d outside the body. This means that the integral equation (3.2) can

in principle be used to determine the potential ϕ in the entire solution domain d+b, for instance by using an iterative procedure based on a recurrence relation such as that given by equation (3.6). As a matter of fact, such an approach may be necessary in the case of subsonic compressible flows governed by a weakly-nonlinear equation of the form $\nabla^2\phi=g(\phi)$. However, in the case of incompressible flows it would usually be much simpler to solve for ϕ on the body surface b, and -- in the event (rare in reality) that knowledge of ϕ outside b is in fact required -- to determine ϕ outside b by means of equation (2.5a), which here takes the simplified form

$$\phi_* = -\int_b G\phi_n da + \int_b \phi G_n da \quad . \tag{3.4}$$

Equation (3.2) also holds for $\vec{\xi}$ inside the body. It thus might appear that this integral equation must also define the potential ϕ inside the body. This result, were it true, would certainly be quite surprising, indeed fundamentally unacceptable, for it would mean that the "exterior boundary-value problem" defined by the Laplace equation in the exterior domain d and a Neumann condition on b would define a solution in the interior domain d_i. It can easily be shown however, that equation (3.2) allows the potential ϕ to be extended inside b in an entirely arbitrary manner. Indeed, equation (3.1) can be written in the form

$$C\phi_* = -\int_b G\phi_n da + \int_b \phi G_n da \quad , \tag{3.5}$$

where C is given by $C=1+\int_b G_n da$. We have $C\equiv 0$ for $\vec{\xi}$ inside b, as may easily be verified (and indeed has already been shown in the previous section), so that equation (3.5) clearly does not define ϕ_* inside b. For $\vec{\xi}$ outside b, on the other hand, we have $C\equiv 1$, and equation (3.5) becomes equation (3.4), which clearly defines ϕ_*.

An interesting feature distinguishing the Landweber integral equation (3.2) from the classical integral equation (3.3) is that whereas the dipole integral in the usual integral equation (3.3) is discontinuous at the body surface b (as was discussed in the previous section, and is indeed well known), the corresponding integral in equation (3.2) is a continuous function (the dipole strength $\phi-\phi_*$ vanishes as the point of integration $\vec{x}$ and the field

point $\vec{\xi}$ coincide). As a matter of fact, the factor 1/2 on the left side of equation (3.3) is correct only at points $\vec{\xi}$ where b is smooth (that is, has a tangent plane), as was also discussed in the previous section. The classical integral equation (3.3) thus requires evaluation of a discontinuous function exactly on the surface of discontinuity of that function. This awkward problem, notably from the point of view of numerical calculations, is avoided in the integral equation (3.2). As a matter of fact, the integrand $(\phi-\phi_*)G_n$ in the integral $L(\vec{\xi};\phi)$ defined by equation (3.2b) is non-singular (i.e. remains finite) as $\vec{x} \to \vec{\xi}$ (as can easily be verified, and indeed is shown in Chow, Hou, and Landweber), which evidently is advantageous for purposes of numerical calculations; for two-dimensional flows, the integrand $(\phi-\phi_*)G_n$ actually vanishes as $\vec{x} \to \vec{\xi}$.

A choice of methods is available for solving the nonhomogeneous Fredholm integral equation (3.2). In particular, a natural method of solution consists in using an iterative procedure. An obvious recurrence relation is that obtained by simply replacing ϕ by $\phi^{(k)}$ and $\phi^{(k+1)}$ on the right and left sides, respectively, of equation (3.2), that is

$$\phi^{(k+1)}(\vec{\xi}) = f(\vec{\xi}) + L(\vec{\xi};\phi^{(k)}), \quad k \geq 0 \quad , \tag{3.6}$$

where the initial (zeroth) approximation $\phi^{(0)}$ must be specified somehow. This recurrence relation has in fact been used by Chow, Hou, and Landweber (1976), equation (20), with the initial approximation $\phi^{(0)}(\vec{\xi})$ taken as twice the given term $f(\vec{\xi})$. The practical usefulness of such an iterative method of solution depends on fast convergence of the successive iterative approximations $\phi^{(k)}$, which in turn depends on selection of a reasonably-good initial approximation $\phi^{(0)}$. Various initial approximations and associated iterative approximations will be investigated in this study.

4. Potential flow about ellipses and ellipsoids

In this section, the integral equation (3.2) and the associated recurrence relation (3.6) are investigated by considering the simple cases of potential flows due to translatory motions of elliptical cylinders and ellipsoids. Two-dimensional potential flows due to translatory motions of an ellipse in the directions of its major and minor axes are considered first. The ellipse is defined by the equation $x^2+y^2/b^2=1$, where b is the thickness ratio of the ellipse (b=thickness/chord). Both "longitudinal" and "transversal" motions of the ellipse with unit velocity in the x and y directions of the major and minor axes, respectively, are considered.

For longitudinal motion, the velocity potential on the surface of the ellipse is given by

$$\phi = -bx \; . \tag{4.1}$$

Furthermore, it can be shown that the given term f defined by equation (3.2a) actually is proportional to the potential ϕ; specifically, we have

$$f/\phi = 1/(1+b) \; . \tag{4.1a}$$

Use of equation (4.1a) in the integral equation (3.2) then yields

$$L(\phi)/\phi = b/(1+b). \tag{4.1b}$$

For transversal motion, on the other hand, we have

$$\phi = -(1-x^2)^{1/2} \tag{4.2}$$

on the upper portion ($y\geq 0$) of the surface of the ellipse. The terms f and $L(\phi)$ in the integral equation (3.2) are also proportional to ϕ; specifically, we have

$$f/\phi = b/(1+b) \quad , \quad L(\phi)/\phi = 1/(1+b) \quad . \tag{4.2a,b}$$

It thus can be seen that the value of f/ϕ for longitudinal (transversal) motion of the ellipse is identical to the value of $L(\phi)/\phi$ for transversal (longitudinal) motion.

The functions f/ϕ and $L(\phi)/\phi$ for longitudinal (L) and transversal (T) motions are represented in figure 2 for values of b varying between 0 and 1,

for which the ellipse becomes a flat plate and a circle, respectively. Figure 2 clearly shows that the known term f in the integral equation (3.2) is larger than the unknown term $L(\phi)$ for longitudinal motion of the ellipse, whereas the opposite is true for transversal motion; in the intermediate case of a circle (b=1), we have $f=L(\phi)=\phi/2$. In the particular case of a thin ellipse ($b<<1$), equations (4.1) show that we have

$$\phi = O(b),\ f = O(b),\ L(\phi) = O(b^2) \text{ and } L(\phi) << f \simeq \phi \tag{4.3a}$$

for longitudinal motion, whereas for transversal motion equations (4.2) yield

$$\phi = O(1),\ f = O(b),\ L(\phi) = O(1) \text{ and } f << L(\phi) \simeq \phi \ . \tag{4.3b}$$

The given term f in the integral equation (3.2) may thus be used as a "thin-wing approximation" for longitudinal motion of the ellipse, but clearly not for transversal motion.

Three-dimensional potential flows due to translatory motions of an ellipsoid are now considered. The ellipsoid is defined by the equation $x^2+y^2/\beta^2+z^2/\beta^2\gamma^2=1$, where the parameters β and γ may take values between 0 and 1, that is $0\leq\beta\leq 1$ and $0\leq\gamma\leq 1$. Translatory motions of the ellipsoid with unit velocity in the x,y, and z directions of the major, intermediate, and minor axes are considered. The velocity potentials corresponding to these three fundamental translatory motions are denoted by ϕ^x,ϕ^y, and ϕ^z, and the corresponding potentials f given by equation (3.2a) are similarly denoted by f^x,f^y, and f^z.

For translation with unit velocity in the positive x direction, we have $\phi_n=\vec{i}\cdot\vec{n}$, where $\vec{i}$ is the unit vector along the positive x axis, and $\vec{n}$ in the unit inward normal vector to the surface (b) of the ellipsoid as previously defined. Equations (3.2a) and (2.1) then yield

$$f^x(\vec{\xi}) = \frac{1}{4\pi}\int_b \frac{1}{|\vec{x}-\vec{\xi}|}\vec{i}\cdot\vec{n}(\vec{x})da(\vec{x}) \quad .$$

Use of the divergence theorem gives

$$f^x(\vec{\xi}) = \frac{1}{4\pi}\int_{d_i} \frac{\partial}{\partial x}\ \frac{1}{|\vec{x}-\vec{\xi}|}\ dv(\vec{x}) = \frac{-1}{4\pi}\int_{d_i} \frac{\partial}{\partial \xi}\ \frac{1}{|\vec{x}-\vec{\xi}|}\ dv(\vec{x}),$$

where d_i is the domain inside the surface of the ellipsoid. By defining the function $F(\vec{\xi})$ as

$$F(\vec{\xi}) = \frac{-1}{4\pi} \int_{d_i} \frac{1}{|\vec{x}-\vec{\xi}|} dv(\vec{x}) \equiv \int_{d_i} G(\vec{x},\vec{\xi}) dv(\vec{x}) \quad , \tag{4.4}$$

we may finally express the potential f^x in the form

$$f^x(\vec{\xi}) = \partial F(\vec{\xi})/\partial \xi \ . \tag{4.5a}$$

Similarly, we have

$$f^y(\vec{\xi}) = \partial F(\vec{\xi})/\partial \eta \quad , \quad f^z(\vec{\xi}) = \partial F(\vec{\xi})/\partial \zeta \ . \tag{4.5b,c}$$

It is known from Havelock (1931) that the velocity potentials ϕ^x, ϕ^y, and ϕ^z for unit-velocity translations of the ellipsoid along the x,y, and z axes can be expressed in terms of the function $F(\vec{\xi})$ defined by equation (4.4). Specifically, we have

$$\phi^x = (1/\lambda^x)\partial F/\partial \xi, \ \phi^y = (1/\lambda^y)\partial F/\partial \eta, \ \phi^z = (1/\lambda^z)\partial F/\partial \zeta \quad , \tag{4.6a,b,c}$$

where λ^x, λ^y, and λ^z are constants defined by the integrals

$$\lambda^x = 1 - \int_0^\infty \frac{\beta^2 \gamma dt}{2(t+1)E} \ , \ \lambda^y = 1 - \int_0^\infty \frac{\beta^2 \gamma dt}{2(t+\beta^2)E} \ , \ \lambda^z = 1 - \int_0^\infty \frac{\beta^2 \gamma dt}{2(t+\beta^2\gamma^2)E} \ , \tag{4.7a,b,c}$$

in which E is the expression given by $E=[(t+1)(t+\beta^2)(t+\beta^2\gamma^2)]^{1/2}$.

It may then be seen from equations (4.5) and (4.6) that the given term $f(\vec{\xi})$ in the integral equation (3.2) is proportional to the velocity potential $\phi(\vec{\xi})$ for translatory motions of an ellipsoid. Specifically, we have

$$f^x/\phi^x = \lambda^x, \ f^y/\phi^y = \lambda^y, \ f^z/\phi^z = \lambda^z \quad , \tag{4.8a,b,c}$$

where the constants λ^x, λ^y, and λ^z are defined by the integrals (4.7). These constants of proportionality are represented in figure 3 as functions of the

parameters β and γ defining the ellipsoid. This figure is subdivided into four parts.

The lower left part of figure 3 represents λ^x as a function of $\beta(0\leq\beta\leq 1)$ for given values of $\gamma(\gamma=0,.2,.4,.6,.8,1)$. We have $1\geq\lambda^x\geq 2/3$, with $\lambda^x=2/3$ in the limiting case of a sphere ($\beta=1=\gamma$). For translation of an ellipsoid along its major axis, the known term f in the integral equation (3.2) may then be seen to be the dominant term in comparison with the unknown term $L(\phi)$, which does not exceed $\phi/3$. In particular, the value of the constant λ^x is quite close to 1 for small values of β. This shows that the term f provides a fairly good "slender-body approximation" to the velocity potential ϕ for translation of a slender ellipsoid in the direction of its major axis. Comparison between the values of $\lambda=f/\phi$ and of $\lambda^x=f^x/\phi^x$ represented in figures 2 and 3 for longitudinal motions of an ellipse and of an ellipsoid, respectively, show that we have $\lambda(b)<\lambda^x(\beta;\gamma=1)\leq\lambda^x(\beta;0\leq\gamma\leq 1)$ for corresponding values of the thickness and slenderness parameters b and β, respectively. More precisely, we have $1/2\leq\lambda(b)=1/(1+b)\leq 1$, and

$$\frac{2}{3}\leq\lambda^x(\beta;\gamma=1) = \frac{1}{1-\beta^2}\,[1+\frac{\beta^2}{2(1-\beta^2)^{1/2}}\,\ell n\,\frac{1-(1-\beta^2)^{1/2}}{1+(1-\beta^2)^{1/2}}]\leq 1 \quad ,$$

as may be obtained by evaluating the integral (4.7a) in the special case of a spheroid ($\gamma=1$). We thus have, for instance, $\lambda=.833$ for b=.2 and $.944\leq\lambda^x\leq 1$ for $\beta=.2$ and $1\geq\gamma\geq 0$.

The lower right and upper left parts of figure 3 correspond to translation of the ellipsoid in the direction of its intermediate axis, and represent the constant $\lambda^y(\beta,\gamma)=f^y/\phi^y$ both as a function of β for given values of γ (lower right part of figure 3) and as a function of γ for given values of β (upper left part). It may be seen that we have $1\geq\lambda^y\geq 1/2$, with $\lambda^y=1/2$ in the limiting case $\beta=0,\gamma=1$ corresponding to a circular cylinder. Finally, the upper right part of figure 3 corresponds to translation of the ellipsoid along its minor axis, and represents $\lambda^z(\beta,\gamma)=f^z/\phi^z$ as a function of γ for given values of β. In this case we have $0\leq\lambda^z\leq 2/3$, with $\lambda^z=2/3$ for $\beta=1=\gamma$ (sphere). In summary, the given term f in the integral equation (3.2) may be seen to be dominant for translation of the ellipsoid in the directions of its major and minor axes, whereas for translation along its minor axis the term $L(\phi)$ is usually dominant, except for values of γ approximately given by $1-\beta/2\leq\gamma\leq 1$.

We conclude this investigation of the integral equation (3.2) for flows due to translatory motions of an ellipsoid by examining the iterative procedure defined by the recurrence relation (3.6) and the initial approximation $\phi^{(0)} \equiv 0$. By using the fact that we have $f=\lambda\phi$, we may obtain $\phi^{(1)}=f, \phi^{(2)}=[1+(1-\lambda)]f, \phi^{(3)}=[1+(1-\lambda)+(1-\lambda)^2]f$, and more generally $\phi^{(n)}=[1+(1-\lambda)+\ldots+(1-\lambda)^{n-1}]f$ for $n \geq 1$. This series converges to $f/[1-(1-\lambda)] \equiv f/\lambda \equiv \phi$ as $n \to \infty$ provided $-1<1-\lambda<1$. We have $1 \geq \lambda \geq 0$, which yields $0 \leq 1-\lambda \leq 1$. The above-defined series and associated iterative procedure therefore converge, except in the limiting case $\lambda=0$ corresponding to translation of an elliptical disk ($\gamma=0$) in the direction normal to its plane (direction of the minor axis of the degenerate ellipsoid).

More precisely, the relative error, ε_n say, associated with the $n^{\underline{th}}$ iterative approximation $\phi^{(n)}$, defined as $\varepsilon_n=(\phi-\phi^{(n)})/\phi$, can be shown to be given by $\varepsilon_n=(1-\lambda)^n$. The number of iterations n required for obtaining an approximation to the potential within a specified relative error ε then is given by $n \geq \ell n\varepsilon/\ell n(1-\lambda)$. For instance, for $\varepsilon=10^{-2}$ the following number of iterations is required

λ	.9	.8	.7	.6	.5	.4	.3	.2	.1
n	2	3	4	5	7	9	13	21	44

depending on the value of λ. As an application, we consider translatory maneuvering motions of an ellipsoidal ship hull form with values of β and γ equal to .15 and .7, corresponding to the fairly typical values of the beam/length and draft/length ratios .15 and .05, respectively. Figure 3 indicates corresponding values of $\lambda^x=f^x/\phi^x$ and $\lambda^y=f^y/\phi^y$ approximately equal to .975 and .6, for which we need 2 and 5 iterations, respectively.

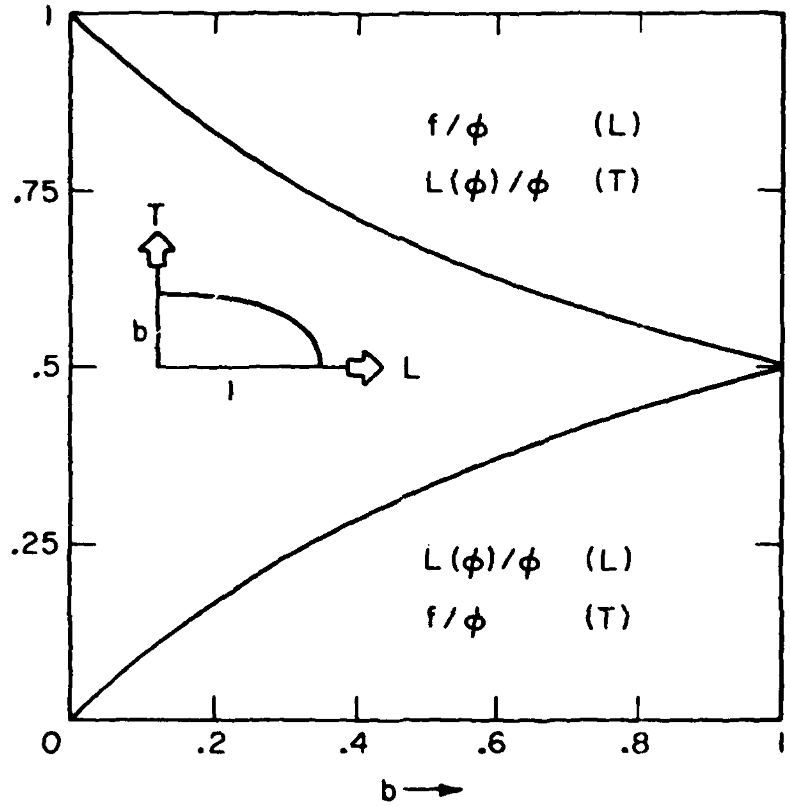

Figure 2 - The ratios f/ϕ and $L(\phi)/\phi$ for longitudinal and transversal translation of ellipses

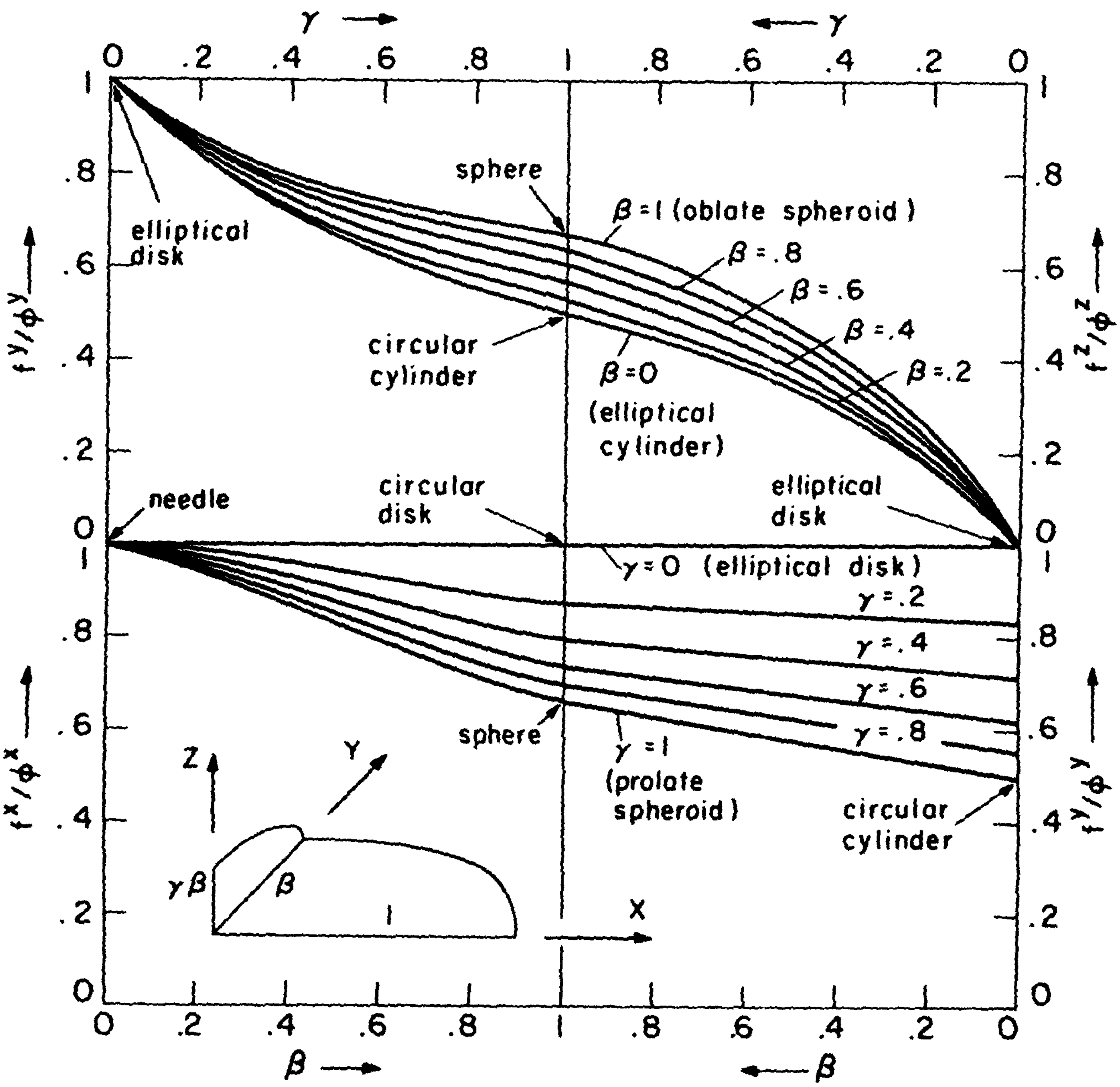

Figure 3 - The ratios f^x/ϕ^x, f^y/ϕ^y, and f^z/ϕ^z for translation of ellipsoids along their major (x), intermediate (y) and minor (z) axes

5. Potential flow about ogives

The remarkable property, enjoyed by ellipsoids, that the given term f in the integral equation (3.2) is proportional to the velocity potential ϕ does not hold in general of course. It thus is useful to consider other body shapes besides ellipsoids. For simplicity, calculations have been performed for two-dimensional potential flows due to translations of ogives along their major and minor axes. A main reason for considering flows due to translations of ogives is that the velocity potential ϕ can be determined analytically, by using conformal mapping. Another recommendation of ogives is that their pointed shapes obviously supplement ellipses.

The ogives are defined by the equation

$$x^2 + [|y|+(1-b^2)/2b]^2 = [(1+b^2)/2b]^2 \quad , \tag{5.1}$$

where b is the thickness ratio ($0\leq b\leq 1$) and $-1\leq x\leq 1$, or by the parametric equations

$$|x| = \frac{1-t^2}{1+t^2+2t(1-b^2)/(1+b^2)} \quad , \quad |y| = \frac{4bt/(1+b^2)}{1+t^2+2t(1-b^2)/(1+b^2)} \quad , \tag{5.1a,b}$$

where t is defined as $t=(\tan\frac{\lambda}{2})^{\delta}$ with $\delta=2(1-\frac{2}{\pi}\tan^{-1}b)$, and the parameter λ varies between the values 0 and $\pi/2$ (we have $|x|=1$ and $y=0$ for $\lambda=0$, and $x=0$ and $|y|=b$ for $\lambda=\pi/2$). In the limit $b=1$, the ogive becomes the unit circle $x^2+y^2=1$, while in the limit $b\to 0$ we have $|y|\sim b(1-x^2)$.

The velocity potentials, on the body surface b, due to unit-velocity translations of ellipses, defined by the equation $|y|=b(1-x^2)^{1/2}$, and ogives with corresponding values of the thickness ratio b are represented in figures 4 and 5 for longitudinal and transversal motions (translations along the major and minor axes), respectively. These figures represent the potential ϕ for x varying between the forward stagnation point and the midchord point, that is for $1\geq x\geq 0$ and $0\leq x\leq 1$ in figures 4 and 5, respectively. The potential ϕ on the surface of an ellipse in longitudinal motion is a linear function of x; more precisely we have $\phi=-bx$, as is given by equation (4.1). The potential on the surface of an ogive is given by

$$\phi = x-(\cos\lambda)/(1-\frac{2}{\pi}\tan^{-1}b) \quad , \tag{5.2a}$$

where $x(0\leq x\leq 1)$ is given by equation (5.1a), and $0\leq\lambda\leq\pi/2$. This potential is

not a monotonic function of x, as may be seen from figure 4. The potential ϕ on the upper surface ($y>0$) of an ellipse in transversal motion is given by $\phi=-(1-x^2)^{1/2}$, for any value of the thickness ratio b. For transversal motion of an ogive, we have

$$\phi = y - (\sin\lambda)/(1 - \frac{2}{\pi}\tan^{-1}b) \quad , \tag{5.2b}$$

where $y(b\geq y\geq 0)$ is given by equation (5.1b), and $0\leq\lambda\leq\pi/2$. This potential becomes identical to the potential $\phi=-(1-x^2)^{1/2}$ for an ellipse in the limiting cases b=1 and b=0 (corresponding to flow past a circle and a normal flat plate, respectively, as is indicated in figure 5.

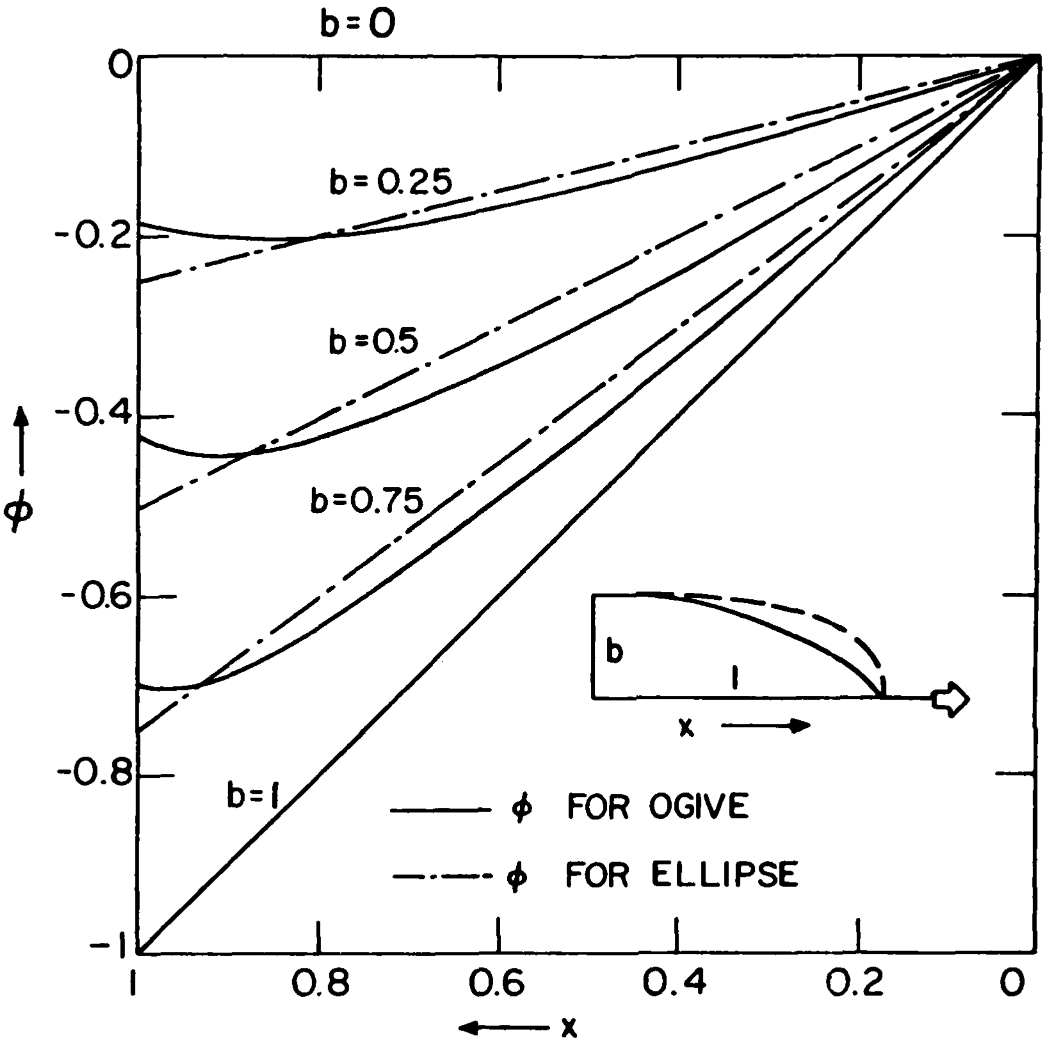

Figure 4 - The velocity potential ϕ on the surfaces of ellipses and ogives in longitudinal translation

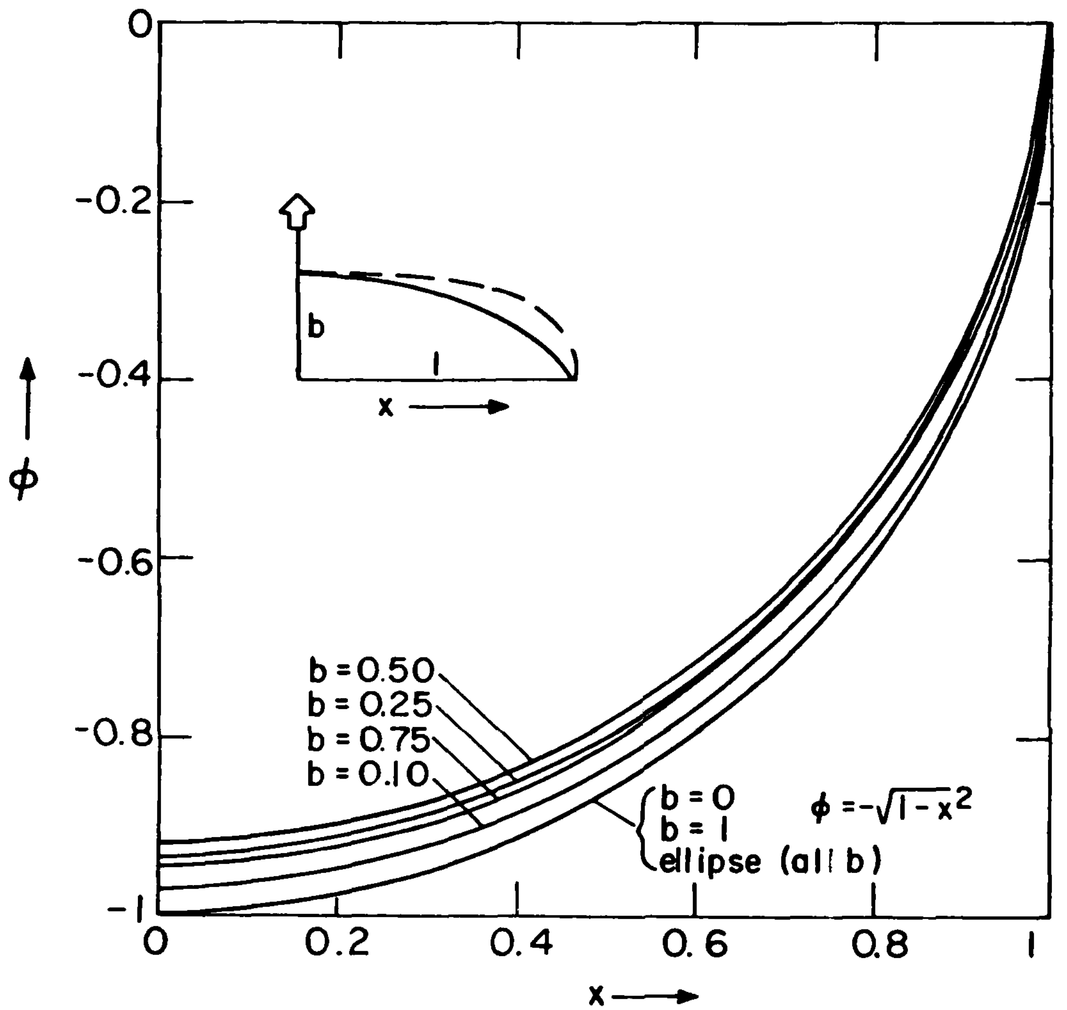

Figure 5 - The velocity potential ϕ on the surfaces of ellipses and ogives in transversal translation

6. Simple explicit approximations for the potential

It was shown in section 4, equations (4.1a) and (4.2a), that the velocity potential ϕ on the surface of an ellipse, with thickness ratio b, is related to the given term f in the integral equation (3.2) by the equations

$$\phi = (1+b)f \quad \text{and} \quad \phi = (1+1/b)f \tag{6.1a,b}$$

for longitudinal and transversal translations, respectively. This result suggests that useful simple explicit approximations to the velocity potential for longitudinal and transversal translations of other cylinders might be given by the potentials $(1+b)f$ and $(1+1/b)f$, respectively, where b is the thickness ratio of the cylinder and f is defined by formula (3.2a).

In particular, these explicit approximations might be used as zeroth-order (initial) approximation $\phi^{(0)}$ in the recurrence relation (3.6), that is

$$\phi^{(0)} = (1+b)f \quad \text{and} \quad \phi^{(0)} = (1+1/b)f \tag{6.2a,b}$$

for longitudinal and transversal motions, respectively. The first-order iterative approximations $\phi^{(1)}$ given by equations (3.6) and (6.2) are

$$\phi^{(1)} = f + (1+b)L(f) \quad \text{and} \quad \phi^{(1)} = f + (1+1/b)L(f) \tag{6.3a,b}$$

for longitudinal and transversal motions, respectively. These approximations are also exact for translations of elliptical cylinders, as may be verified by using equations (6.1a,b) in the term $L(\phi)$ on the right side of the integral equation (3.2).

The approximations (6.2b) and (6.3b) for transversal translation lead to numerical difficulties in the limiting case of thin cylinders, that is for $b \ll 1$. Indeed, we have $f \to 0$ as $b \to 0$, so that the terms f/b and $L(f)/b$ are in the indeterminate form 0/0. An alternative to these approximations, for the case of transversal translation of thin cylinders, consists in using the potential, ϕ_{fp} say, for flow past a normal flat plate as zeroth-order approximation $\phi^{(0)}$. This then yields the zeroth-and first-order approximations

$$\phi^{(0)} = \phi_{fp} = -(1-x^2)^{1/2} \text{ for } y > 0, \text{ and } \phi^{(1)} = f + L(\phi_{fp}). \tag{6.4a,b}$$

These approximations again are exact in the particular case of elliptical cylinders, as may readily be seen from equations (4.2) and (3.2).

The above-defined approximations are compared to the exact potential ϕ on the surfaces of ogives in longitudinal and transversal translations in figures 6, 7, and 8. Figure 6 shows the potential ϕ and the approximations f, (1+b)f, and f+(1+b)L(f) for longitudinal motion of thin ogives, with thickness ratio b equal to .1, .2 and .3. This figure clearly shows that multiplication of the basic thin-body approximation f by the constant (1+b) yields significant improvements, and the improvements obviously are more important for larger values of b. Figure 6 also depicts the improvements of the zeroth-order approximation $\phi^{(0)}=(1+b)f$ due to the first-order approximation $\phi^{(1)}=f+(1+b)L(f)$. These improvements are apparent also from figure 7, in which the approximations (1+b)f and f+(1+b)L(f) are compared to the exact potential ϕ for longitudinal translation of ogives with thickness ratio b equal to .25, .5, .75, and 1. For b=1, the ogive is a circle and the approximations (1+b)f and f+(1+b)L(f) are identical to $\phi\equiv-x$.

Figure 8 corresponds to transversal translation of ogives and represents the potential ϕ and the approximations (1+1/b)f, f+(1+1/b)L(f) and $f+L(\phi_{fp})$ for b=0, .1, .25, .5, .75, and 1; the zeroth-order approximation $\phi_{fp}=-(1-x^2)^{1/2}$ has already been represented, together with the potential ϕ, in figure 5. In the limiting case b=1, all these approximations are identical to the exact potential ϕ, which is given by $\phi=-(1-x^2)^{1/2}$. For fairly large values of b, say for b=.75 and for b=.5, the zeroth-and first-order approximations (1+1/b)f and f+(1+1/b)L(f) may be seen to be fairly close to the potential ϕ, and somewhat superior to the alternative approximations ϕ_{fp} and $f+L(\phi_{fp})$. Figure 8 also shows that the first approximation f+(1+1/b)L(f) improves upon the zeroth approximation (1+1/b)f. The approximations (1+1/b)f and f+(1+1/b)L(f) may however be seen to deteriorate as b decreases, and the alternative approximations ϕ_{fp} and $f+L(\phi_{fp})$ clearly are superior for fairly small values of b, say for $b\leq.25$. In particular, in the limiting case of a flat plate (b=0), the approximations ϕ_{fp} and $f+L(\phi_{fp})\equiv L(\phi_{fp})$ become exact, equal to the potential $\phi=-(1-x^2)^{1/2}$. On the other hand, we have $(1+1/b)f\sim f/b$ and $f+(1+1/b)L(f)\sim L(f/b)\sim f/b$ as $b\to 0$, so that the approximations (1+1/b)f and f+(1+1/b)L(f) become identical in the limit b=0; however, the limit of f/b as $b\to 0$ is not equal to the potential ϕ, as may be seen from figure 8.

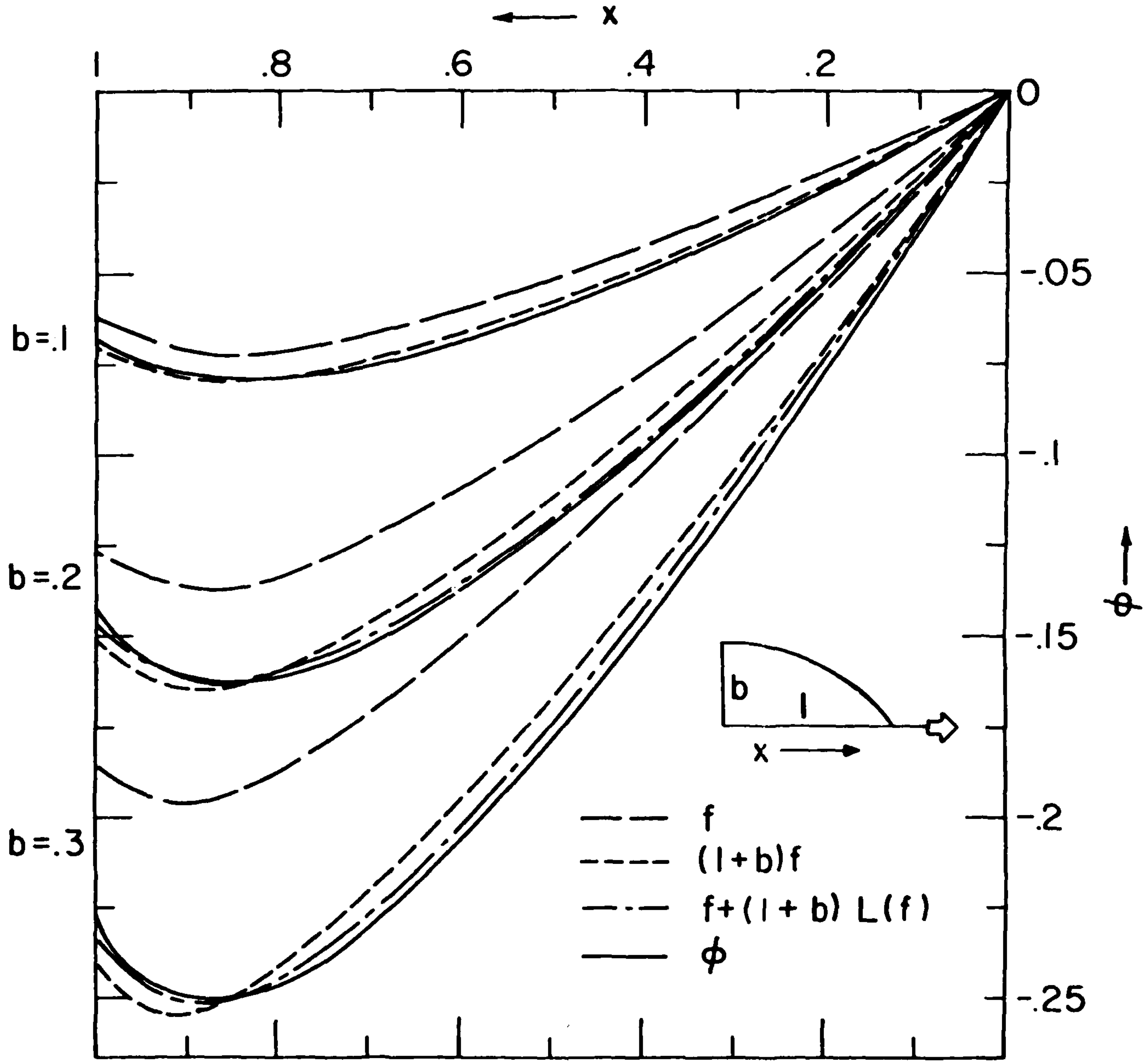

Figure 6 - The potential ϕ and the approximations $(1+b)f$ and $f+(1+b)L(f)$ on the surfaces of thin ogives in longitudinal translation

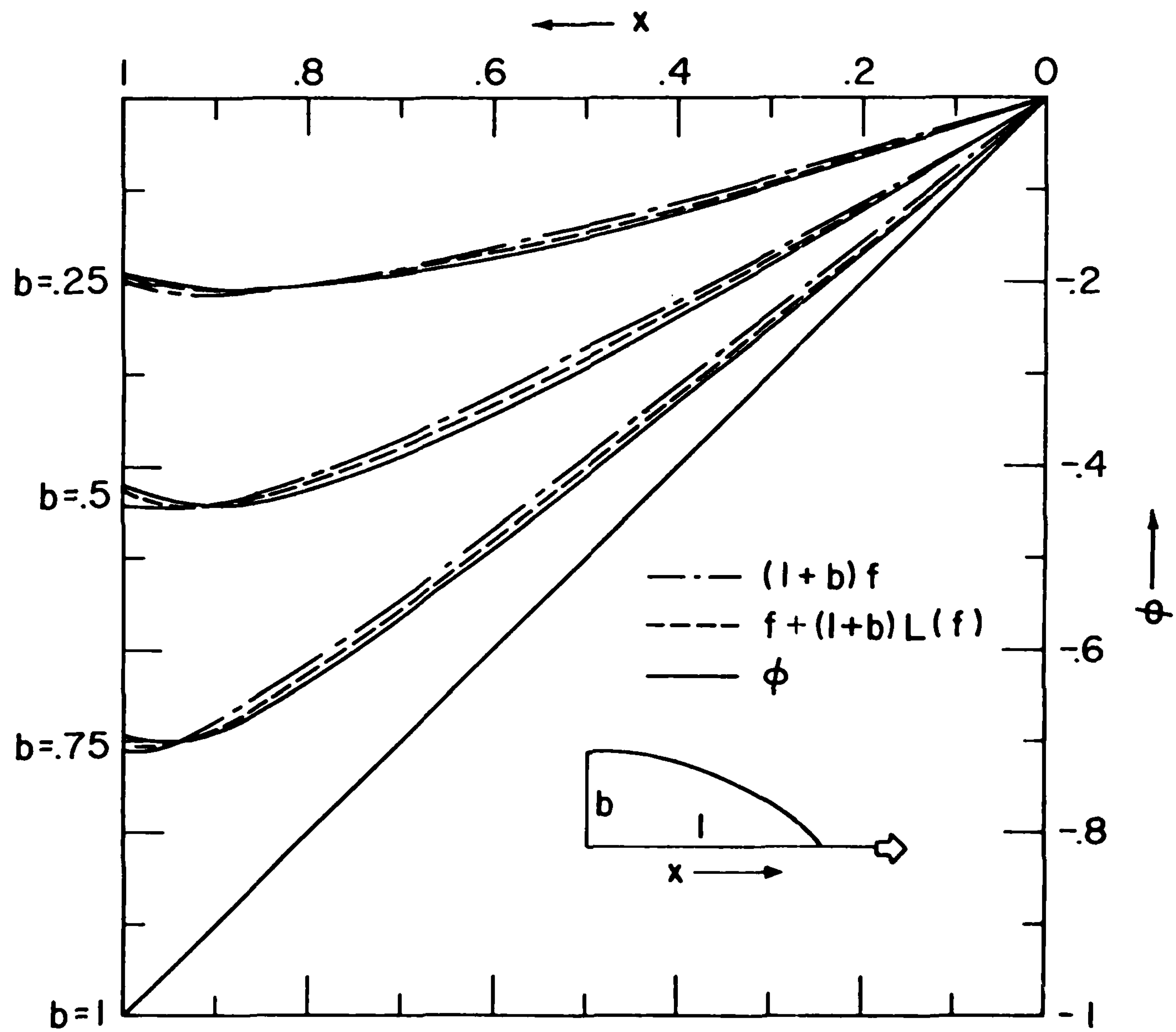

Figure 7 - The potential ϕ and the approximations (1+b)f and f+(1+b)L(f) on the surfaces of ogives in longitudinal translation

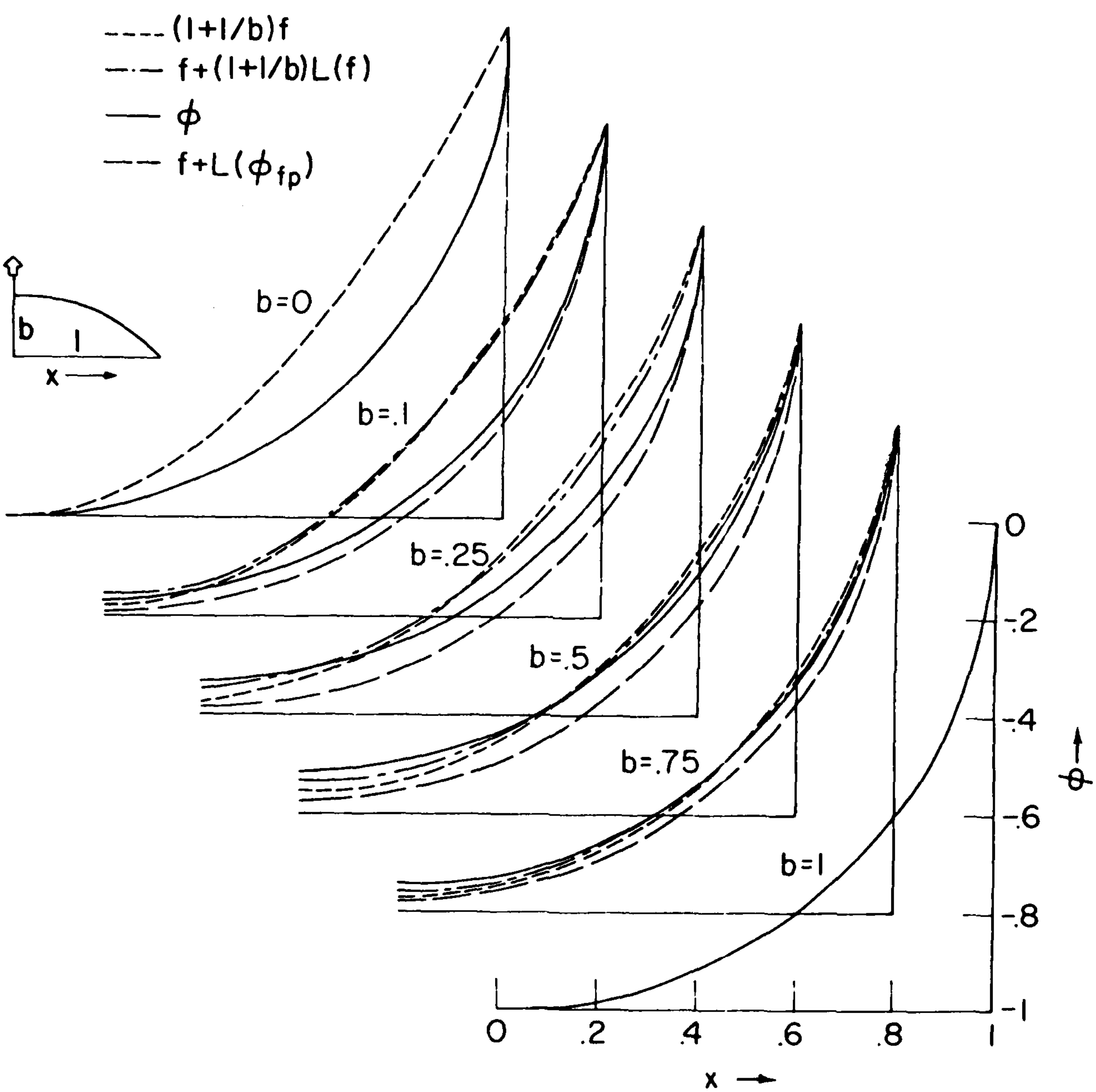

Figure 8 - The potential ϕ and the approximations (1+1/b)f, f+(1+1/b)L(f), and f+L(ϕ_{fp}) on the surfaces of ogives in transversal translation

7. A modified integral equation and related explicit approximation

The preceding investigation of flows due to translations of ellipsoids and ogives suggests that it may be advantageous to express the solution $\phi(\vec{\xi})$ of the integral equation (3.2) in the form $\phi(\vec{\xi})=k(\vec{\xi})f(\vec{\xi})$, where $f(\vec{\xi})$ is the potential given by formula (3.2a) and $k(\vec{\xi})$ is the function defined as $k=\phi/f$. Figures 6, 7, and 8 for flows due to translations of ogives suggest that $k(\vec{\xi})$ may generally be expected to be a slowly varying function. This function in fact is a constant in the particular cases of translations of ellipsoids, as was shown previously and is specifically indicated by equations (4.8). More generally, we will express the potential $\phi(\vec{\xi})$ in the form $\phi(\vec{\xi})=k(\vec{\xi})\psi(\vec{\xi})$, where $\psi(\vec{\xi})$ is some supposedly-given potential. An obvious choice for ψ is the given term f in the integral equation (3.2), as was just discussed. Other choices for the potential ψ can however be made. For instance, for two-dimensional flow due to transversal translation of a thin cylinder, the potential ψ could be taken as the potential $\phi_{fp}=-(1-x^2)^{1/2}$ for flow past a normal flat plate, as was discussed in the previous section.

By expressing the term $\phi(\vec{x})-\phi(\vec{\xi})\equiv k(\vec{x})\psi(\vec{x})-k(\vec{\xi})\psi(\vec{\xi})$ in equation (3.2b) in the form $k(\vec{\xi})[\psi(\vec{x})-\psi(\vec{\xi})]+[k(\vec{x})-k(\vec{\xi})]\psi(\vec{x})$, the integral equation (3.2) becomes

$$\phi(\vec{\xi}) = f(\vec{\xi}) + k(\vec{\xi}) \int_b [\psi(\vec{x})-\psi(\vec{\xi})]G_n da + \int_b [k(\vec{x})-k(\vec{\xi})]\psi(\vec{x})G_n da.$$

Multiplication of this equation by $\psi(\vec{\xi})$ and use of the relation $k\psi=\phi$ then yields

$$\phi(\vec{\xi})\left[\psi(\vec{\xi}) - \int_b [\psi(\vec{x})-\psi(\vec{\xi})]G_n da\right] = f(\vec{\xi})\psi(\vec{\xi}) + \int_b [\phi(\vec{x})\psi(\vec{\xi})-\phi(\vec{\xi})\psi(\vec{x})]G_n da .$$

We thus may obtain the modified integral equation

$$\phi(\vec{\xi}) = \Phi(\vec{\xi}) + L'(\vec{\xi};\phi) \quad , \tag{7.1}$$

where the potential $\Phi(\vec{\xi})$ is defined as

$$\Phi(\vec{\xi}) = f(\vec{\xi})\psi(\vec{\xi})/[\psi(\vec{\xi})-L(\vec{\xi};\psi)] , \tag{7.1a}$$

with $L(\vec{\xi};\psi) = \int_b [\psi(\vec{x})-\psi(\vec{\xi})]G_n(\vec{x},\vec{\xi})da(\vec{x})$ as is given by equation (3.2b), and the term $L'(\vec{\xi};\phi)$ is the linear transform of ϕ defined by the integral

$$L'(\vec{\xi};\phi) = \int_b [\phi(\vec{x})\psi(\vec{\xi})-\phi(\vec{\xi})\psi(\vec{x})]G_n da\Big/[\psi(\vec{\xi})-L(\vec{\xi};\psi)] \quad . \tag{7.1b}$$

The term $L'(\vec{\xi};\phi)$ obviously vanishes if the function $\psi(\vec{\xi})$ is proportional to the potential $\phi(\vec{\xi})$, whereas the function $\varphi(\vec{\xi})$ then becomes identical to $\phi(\vec{\xi})$, as may be verified by using the integral equation (3.2) in expression (7.1a).

It then follows from equations (4.8) that the potential φ defined by equation (7.1a) with ψ taken as the term f given by formula (3.2a), that is the potential φ given by

$$\varphi(\vec{\xi}) = f^2(\vec{\xi})/[f(\vec{\xi})-L(\vec{\xi};f)] \quad , \tag{7.2}$$

is exact in the particular cases of potential flows due to translations of ellipsoids. In general, formula (7.2) defines an explicit approximation for the velocity potential ϕ. This approximation is compared to the exact potential ϕ on the surfaces of ogives in longitudinal and transversal translations in figures 9, 10, and 11.

Figure 9 shows the potential ϕ and the approximations $f, f+L(f)$, and $f^2/[f-L(f)]$ for longitudinal motion of thin ogives, with thickness ratio b= .1, .2, and .3. The approximations f and f+L(f) are the first and second iterative approximations associated with the straightforward recurrence relation (3.6) and the initial approximation $\phi^{(0)}\equiv 0$. Computationally, the approximations f+L(f) and $f^2/[f-L(f)]$ are equivalent. However, the approximation $f^2/[f-L(f)]$ may be seen to be superior to the approximation f+L(f). The superiority of the approximation $f^2/[f-L(f)]$ in comparison with the approximation f+L(f) is apparent also from figure 10, where these two approximations are compared to the exact potential ϕ for longitudinal translation of ogives with thickness ratio b=.25, .5, .75, and 1. In particular, the approximation $f^2/[f-L(f)]$ is identical to the potential $\phi=-x$ in the limiting case b=1, corresponding to a circular cylinder; for b=.75, this approximation is practically indistinguishable from ϕ on the scale of figure 10.

It is also interesting to compare the approximation $f^2/[f-L(f)]$ to the (computationally-equivalent) approximation f+(1+b)L(f). Comparison of figures 10 and 9 to figures 7 and 6, respectively, show that the approximation $f^2/[f-L(f)]$ is superior to the approximation f+(1+b)L(f) except for sufficiently-thin ogives, say for $b\le .2$, for which the two approximations are comparable. Finally, figures 10,9,7, and 6 incidentally provide a comparison between the straightforward second approximation f+L(f) and the modified first approximation (1+b)f. Figures 6 and 9 show that the approximation f+L(f) is somewhat superior to the approximation (1+b)f for b=.1, and that these two approximations are comparable for b=.2 and .3. However, for thicker ogives, say for $.5\le b\le 1$, figures 7 and 10

show that the (computationally-simpler) approximation $(1+b)f$ in fact is superior to the approximation $f+L(f)$.

Figure 11 corresponds to transversal translation of ogives with thickness ratio $b=0$, .1, .25, .5, .75, and 1. This figure shows the approximation $f\phi_{fp}/[\phi_{fp}-L(\phi_{fp})]$, obtained by selecting the potential ψ in equation (7.1a) as the potential $\phi_{fp}=-(1-x^2)^{1/2}$ of flow past a normal flat plate, and the computationally-equivalent approximation $f+L(\phi_{fp})$, obtained previously in equation (6.4b). In the limiting cases $b=0$ and $b=1$, both of these approximations are identical to the exact potential $\phi=-(1-x^2)^{1/2}$. More precisely, we have $f=\phi/2, \phi_{fp}=\phi, L(\phi_{fp})=\phi/2$, and $\phi_{fp}-L(\phi_{fp})=\phi/2$ for $b=1$, while as $b\to 0$ we have $f\to 0$, $\phi_{fp}\sim\phi$, and $\phi_{fp}-L(\phi_{fp})\sim f$, as may be verified from the integral equation (3.2); the approximation $f\phi_{fp}/[\phi_{fp}-L(\phi_{fp})]$ thus is in the indeterminate form 0/0 for $b=0$, although we have $f/[\phi_{fp}-L(\phi_{fp})]\to 1$ as $b\to 0$. Disregarding this potential numerical difficulty for very thin cylinders, figure 11 shows that the approximation $f\phi_{fp}/[\phi_{fp}-L(\phi_{fp})]$ is superior to the approximation $f+L(\phi_{fp})$.

Figure 11 also shows the approximation $f^2/[f-L(f)]$ for $b=1$, .75, and .5. In the limiting case $b=1$, we have $f=\phi/2$ and $L(f)=\phi/4$, so that we have $f-L(f)=\phi/4$ and the approximation $f^2/[f-L(f)]$ is identical to ϕ. For $b=.75$ and .5, this approximation may be seen to be fairly good, and roughly comparable to the approximation $f\phi_{fp}/[\phi_{fp}-L(\phi_{fp})]$. However, the approximation $f^2/[f-L(f)]$ deteriorates as b decreases, and it is not shown for $b<.5$. The reason for the fact that the approximation $f^2/[f-L(f)]$ is not useful for transversal translation of thin cylinders is that the terms f and $L(f)$ then are comparable, as can easily be shown by considering the particular cases of ellipsoids. For flows due to translations of ellipsoids the given term f in the integral equation (3.2) is proportional to the exact potential ϕ, that is we have $f=\lambda\phi$, as is indicated in equations (4.8). Use of this relation in the integral equation (3.2) then yields $L(f)=\lambda L(\phi)=\lambda(\phi-f)=\lambda(1-\lambda)\phi$, so that we have $f-L(f)=\lambda^2\phi$ and $f^2/[f-L(f)]=\lambda^2\phi^2/\lambda^2\phi=\phi$. The approximation $f^2/[f-L(f)]$ thus is exact, as was indeed noted previously, but numerical difficulties may clearly be expected if $L(f)=\lambda(1-\lambda)\phi$ is comparable to $f=\lambda\phi$, which occurs if λ is small. For practical purposes, it may be preferable not to use the approximation $f^2/[f-L(f)]$ for values of λ less than about .4, for which we have $f=.4\phi$, $L(f)=.24\phi$, and $f-L(f)=.16\phi$. Figure 3 then indicates that the approximation $f^2/[f-L(f)]$ might be used for translations of a body along its major and intermediate axes, as well as along its minor axis if the body is not too flat. More precisely, figure 3 shows that we have $\lambda\le .4$ for translation of an ellipsoid alongs its minor axis if $\gamma\le .65-.3\beta$, approximately.

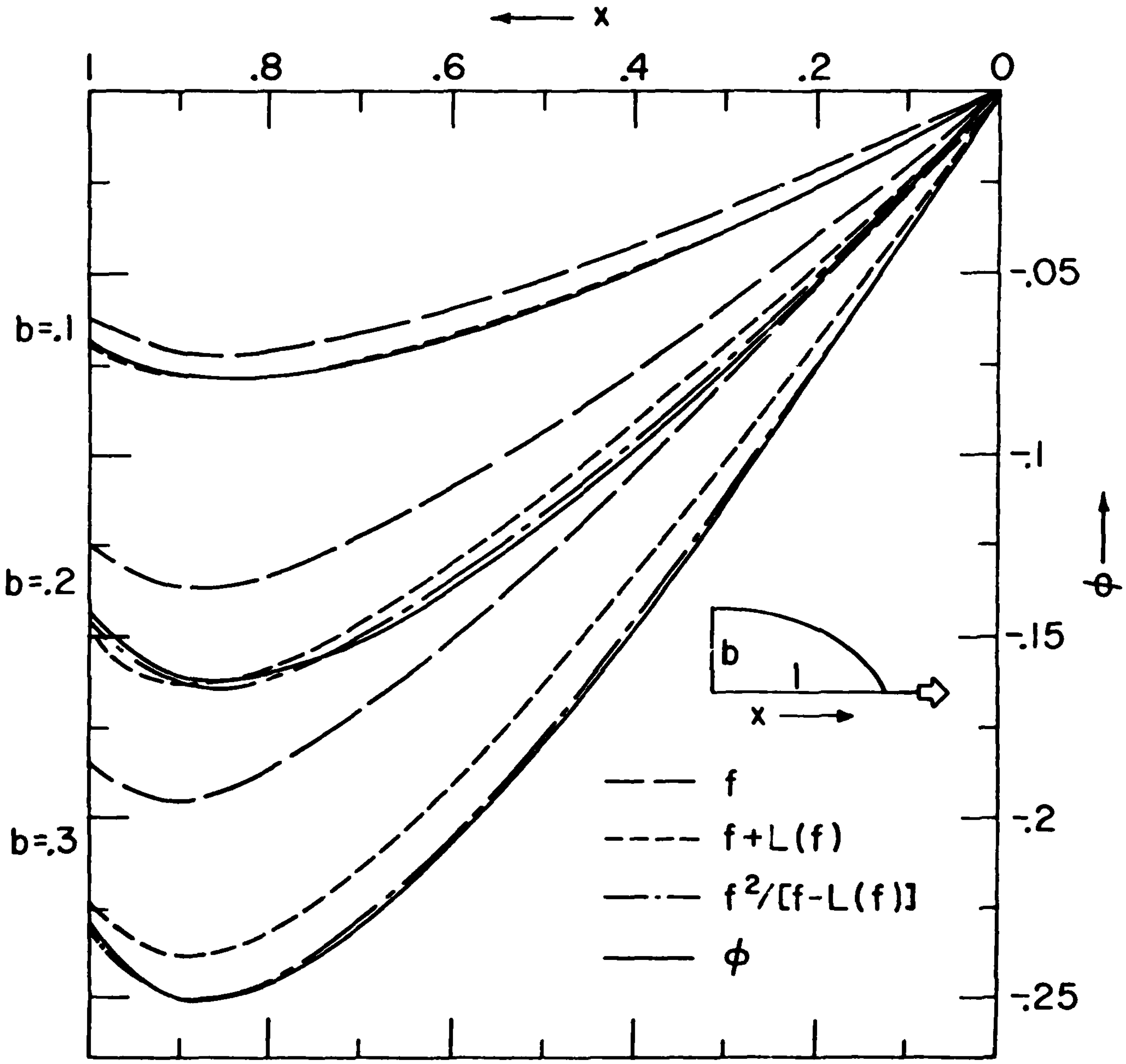

Figure 9 - The potential ϕ and the approximations $f, f+L(f)$, and $f^2/[f-L(f)]$ on the surfaces of thin ogives in longitudinal translation

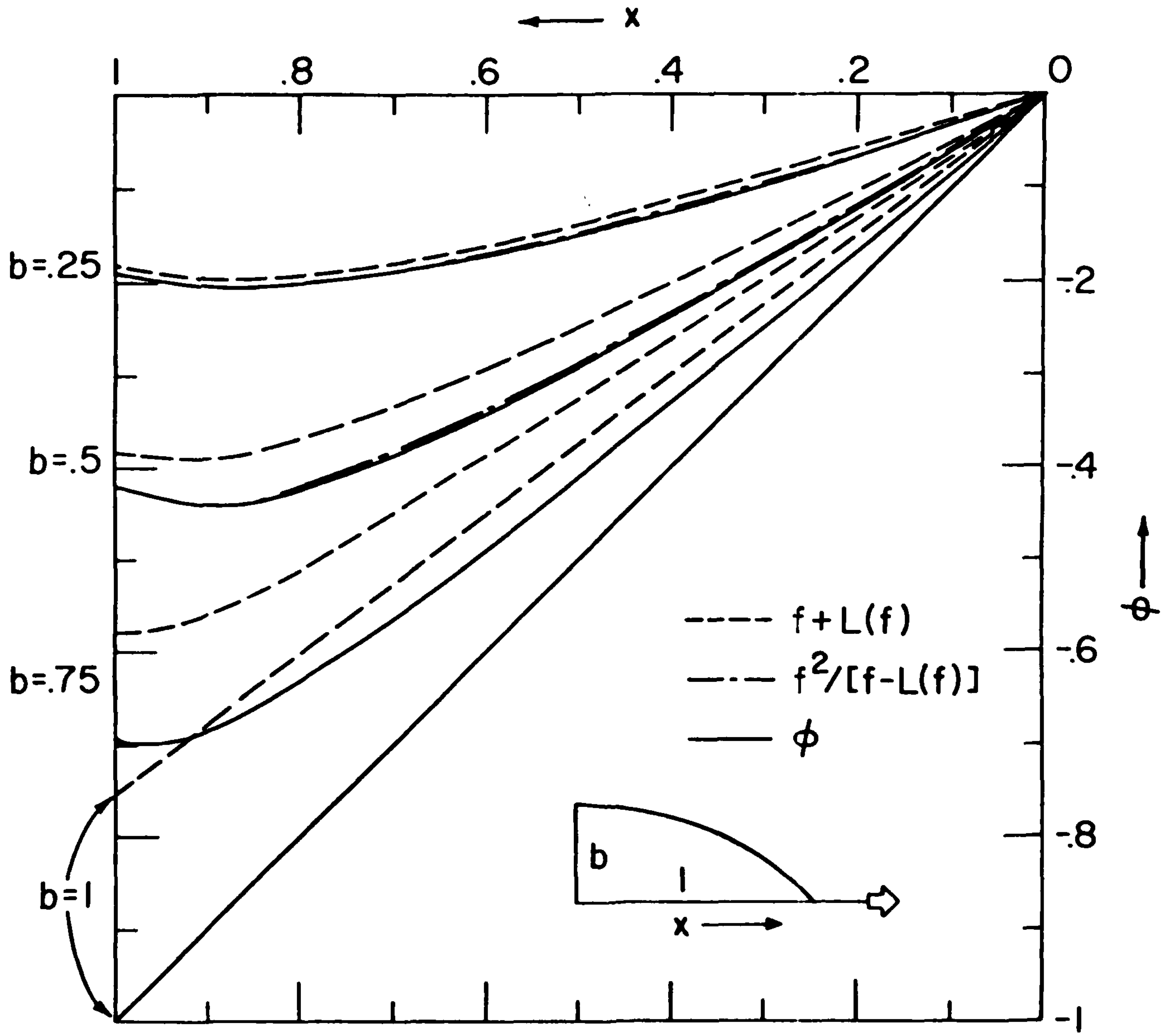

Figure 10 - The potential ϕ and the approximations f+L(f) and $f^2/[f-L(f)]$ on the surfaces of ogives in longitudinal translation

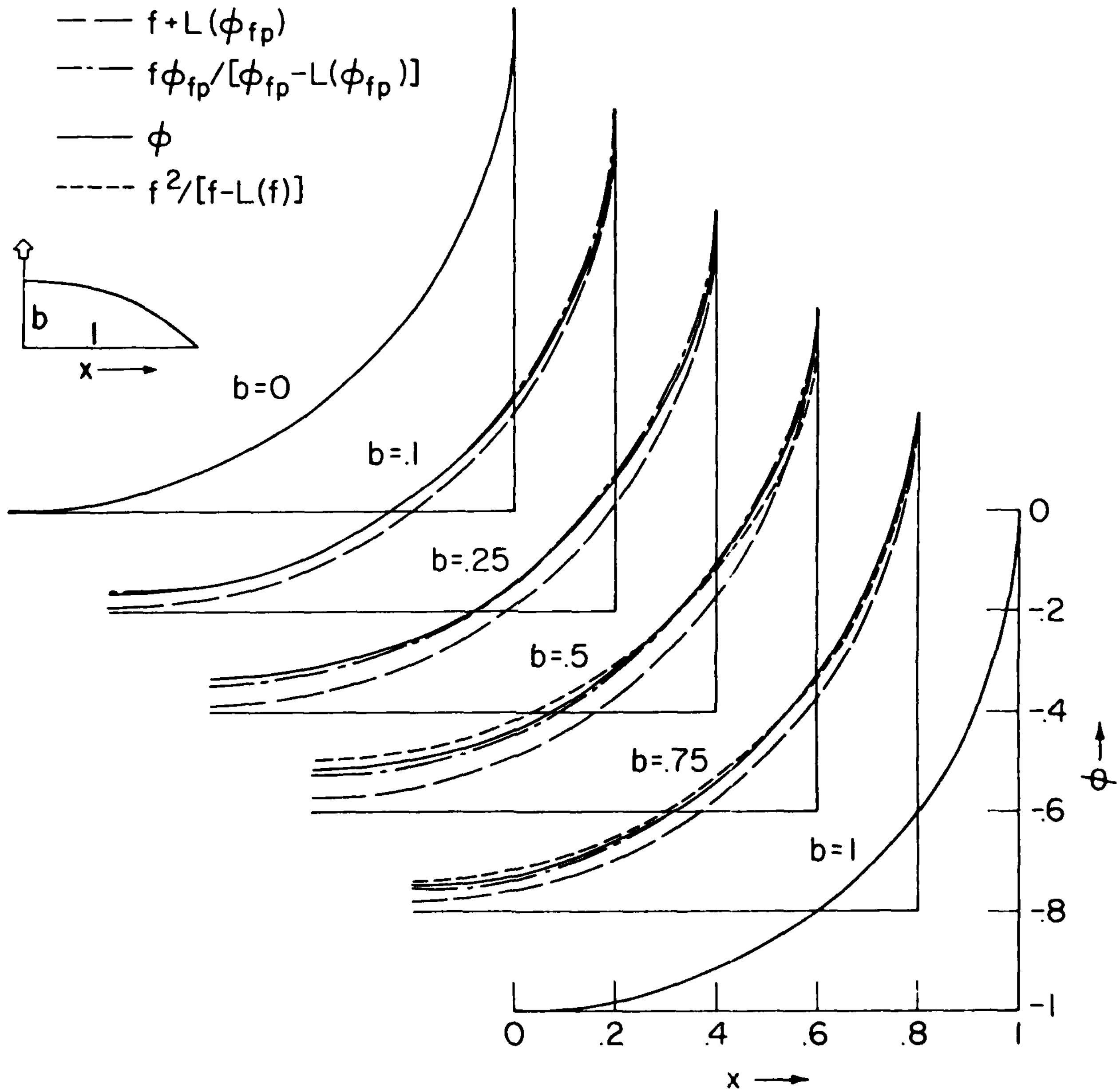

Figure 11 - The potential ϕ and the approximations $f+L(\phi_{fp})$, $f\phi_{fp}/[\phi_{fp}-L(\phi_{fp})]$, and $f^2/[f-L(f)]$ on the surfaces of ogives in transversal translation

8. Conclusion

The foregoing study of flows due to translation of ellipsoids and ogives shows that simple explicit formulas for the velocity potential can provide realistic approximations, which may actually be sufficient for many practical purposes. Two simple explicit approximations appear to be of particular interest. These are the approximation $f(\vec{\xi})$ given by

$$f(\vec{\xi}) = -\int_b G(\vec{x},\vec{\xi})\phi_n(\vec{x})da(\vec{x}) , \tag{8.1}$$

as is defined by formula (3.2a), and the approximation $\varPhi(\vec{\xi})$ given by formulas (7.2) and (3.2b), that is

$$\varPhi(\vec{\xi}) = f^2(\vec{\xi})\Big/[f(\vec{\xi})-\int_b \{f(\vec{x})-f(\vec{\xi})\}G_n(\vec{x},\vec{\xi})da(\vec{x})]. \tag{8.2}$$

The approximation $f(\vec{\xi})$ is a (first-order) slender-body approximation which may be useful for longitudinal translation of a slender body, such as a ship form for instance. As a matter of fact, the approximation (8.1) corresponds to the zero-Froude-number limit of the first-order slender-ship approximation obtained in Noblesse (1978) for the problem of the wave resistance of a ship.

The more complex approximation (8.2) essentially corresponds to a second-order approximation. In fact, in the particular case of longitudinal translation of a slender body, we have $|\int_b[f(\vec{x})-f(\vec{\xi})]G_n da| << |f(\vec{\xi})|$, and the approximation $\varPhi(\vec{\xi})$ is asymptotically equivalent to the straightforward second-order slender-body approximation

$$\phi^{(2)}(\vec{\xi}) = f(\vec{\xi}) + \int_b [f(\vec{x})-f(\vec{\xi})]G_n(\vec{x},\vec{\xi})da(\vec{x}) \tag{8.3}$$

given by the recurrence relation (3.6), with $\phi^{(0)}=0$ (and $\phi^{(1)}=f$). However, the approximation (8.2) has broader applicability, as is suggested by the results of calculations for longitudinal and transversal translation of ogives reported in figures 9, 10, and 11, and by the fact that the potential $\varPhi(\vec{\xi})$ is exact in the particular case of translation of ellipsoids, as was shown previously. Indeed, these results indicate that the explicit approximation $\varPhi(\vec{\xi})$ may be of practical usefulness for a large class of bodies and body motions, excluding however

transversal motions of flat disk-like bodies. More precisely, if $1,\beta$, and δ are the three main dimensions of a body, with $1\geq\beta\geq\delta$, it was predicted that the approximation $\vec{\phi}(\vec{\xi})$ could lead to numerical difficulties for motion in the direction of the minor dimension δ if $\delta\leq(.65-.3\beta)\beta$. This rough tentative criterion for applicability of the approximation $\phi(\vec{\xi})$ does not exclude usual ship hull forms, for instance (typical values of β and δ for double-hull forms are $\beta=.15$ and $\delta=.1$).

Simple explicit approximations analogous to the above approximations (8.1), (8.2), and (8.3) can likewise be obtained for the analogous, although more complex, problems of potential flow about bodies in the presence of a free surface. In particular, first-and second-order slender-ship approximations corresponding to approximations (8.1) and (8.2), respectively, are given in Noblesse (1978) for the problem of the wave resistance of a ship. The integral equation (3.2) and the explicit approximation (8.2) have also been extended to the problem of potential flow about a body in regular water waves in Noblesse (1980).

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Appendix Integral identities for the interior problem and for the combined exterior-interior problems

Although the present study is primarily concerned with the "exterior potential-flow problem", that is the problem of potential flow about a body, it may be interesting to list here the integral identities corresponding to equations (2.5a,b,c) and equation (2.7) for the "interior potential", ϕ^i say, defined in the interior domain d_i. The integral identities corresponding to equations (2.5a,b,c) take the forms

$$\phi_*^i = \int_{d_i} G\nabla^2\phi^i dv + \int_b (G\phi_n^i - \phi^i G_n) da \tag{A1a}$$

for $\vec{\xi}$ in d_i-b, that is inside the surface b,

$$0 = \int_{d_i} G\nabla^2\phi^i dv + \int_b (G\phi_n^i - \phi^i G_n) da \tag{A1b}$$

for $\vec{\xi}$ in d-b, i.e. outside b, and

$$\frac{1}{2}\phi_*^i = \int_{d_i} G\nabla^2\phi^i dv + \int_b (G\phi_n^i - \phi^i G_n) da \tag{A1c}$$

for $\vec{\xi}$ exactly on (smooth)b. The integral identity corresponding to equation (2.7) takes the form

$$0 = \int_{d_i} G\nabla^2\phi^i dv + \int_b [G\phi_n^i - (\phi^i - \phi_*^i)G_n] da \quad . \tag{A2}$$

This integral identity, like equation (2.7), is valid for any point $\vec{\xi}$, either inside, outside, or exactly on the surface b, and indeed is equivalent to the set of the three classical identities (A1a,b,c).

If we add the integral identities (2.5a) and (A1b), we may obtain the relation

$$\phi_* = \int_d G\nabla^2\phi dv + \int_{d_i} G\nabla^2\phi^i dv + \int_b [G(\phi^i_n - \phi_n) + (\phi - \phi^i)G_n]da.$$

Addition of the integral identities (2.5b) and (A1a) yields the same relation, except for the fact that ϕ_* on the left side is replaced by ϕ^i_*. We therefore have the relation

$$\phi_* = \int_{d+d_i} G\nabla^2\phi dv + \int_b [G(\phi^i_n - \phi_n) + (\phi - \phi^i)G_n]da, \tag{A3}$$

where ϕ on the left side and in the first integral on the right side clearly corresponds to ϕ or ϕ^i for points outside or inside the surface b, respectively. Naturally, the integral relation (A.3) can also be obtained by adding identities (2.7) and (A2). Indeed, this yields

$$(1-C_i)\phi_* + C_i\phi^i_* = \int_{d+d_i} G\nabla^2\phi dv + \int_b [G(\phi^i_n - \phi_n) + (\phi - \phi^i)G_n]da \quad ,$$

where $C_i \equiv -\int_b G_n da$. It can easily be seen from equations (2.6) and (2.3) that the expression $(1-C_i)\phi_* + C_i\phi^i_*$ is identical to ϕ_* or ϕ^i_* for $\vec{\xi}$ outside or inside the surface b, respectively, so that the above relation is identical to relation (A3).

The integral relation (A3) expresses the potential $\phi(\vec{\xi})$ in the entire space $d+d_i$ in terms of a volume distribution of sources with density $\nabla^2\phi$, and surface distributions of sources and normal dipoles on the surface b, with densities $\phi^i_n - \phi_n$ and $\phi - \phi^i$, respectively. Two classical results in potential theory immediately follow from relation (A3), namely: (i) a distribution of normal dipoles, with strength δ say, on a surface, S say, generates a potential $\phi = \int_S \delta G_n da$ whose value is discontinuous across the surface S (specifically, we have $\phi^e - \phi^i = \delta$, where superscripts e and i refer to the "exterior" and "interior" sides of S, respectively; the "interior" side being that into which the unit normal vector to S is drawn), and (ii) a distribution of sources, with strength σ say, over a surface S generates a potential $\phi = \int_S G\sigma da$ whose normal derivative ϕ_n is discontinuous across S (specifically, we have $\phi^i_n - \phi^e_n = \sigma$).

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